

OPTIMAL PORTFOLIOS

EXAMPLES

- From Applied:
 - U.S., U.K., France, Germany, Japan
 - U.S., Developed, Emerging
- ETFs from Yahoo Finance

SOLVER IN PYTHON

- There are several choices in python for optimizing functions.
- The qp function from cvxopt is a very good choice for portfolio optimization.
 - cvxopt = convex optimization
 - qp = quadratic programming

MATRIX MULTIPLICATION

PRIMER/REVIEW OF MATRIX MULTIPLICATION

- Excel's MMULT function
- Example:

$$\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 13 \\ 16 \end{pmatrix}$$

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- $(2 \times 2)^* (2 \times 1) = (2 \times 1)$
- height = height of 1st matrix, width = width of 2nd matrix

•
$$(3 \times 2)^* (2 \times 2) = (3 \times 2)$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 2 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 13 & 10 \\ 16 & 12 \\ 19 & 14 \end{pmatrix}$$

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MATRICES IN CVXOPT: PORTFOLIO VARIANCE

• $w^{\top} \operatorname{Cov} w = \operatorname{portfolio} \operatorname{variance} =$

$$egin{array}{cccc} (w_1 & \cdots & w_n) egin{pmatrix} \operatorname{var}_1 & \cdots & \operatorname{cov}_{1n} \ dots & dots & dots & dots & dots \ dots & dots & dots & dots & dots \ dots & dots & dots & dots & dots \ dots & dots & dots & dots & dots \ dots & dots & dots & dots & dots \ dots & dots$$

- $(1 \times n)^* (n \times n)^* (n \times 1) = (1 \times 1)$
- P = covariance matrix in cvxopt notation

MATRICES IN CVXOPT: EQUALITY CONSTRAINTS

• Weights sum to 1 and expected return = target is expressed as:

$$egin{pmatrix} 1 & \cdots & 1 \ \mu_1 & \cdots & \mu_n \end{pmatrix} egin{pmatrix} w_1 \ dots \ w_n \end{pmatrix} = egin{pmatrix} 1 \ \mu_{ ext{target}} \end{pmatrix}$$

• Aw = b in cvxopt notation (Julius should supply A and b)

MATRICES IN CVXOPT: INEQUALITY CONSTRAINTS

• No short sales is equivalent to:

$$egin{pmatrix} -1 & 0 & \cdots & 0 & 0 \ dots & dots & dots & dots & dots & dots \ 0 & 0 & \cdots & 0 & -1 \end{pmatrix}egin{pmatrix} w_1 \ dots \ w_n \end{pmatrix} \leq egin{pmatrix} 0 \ dots \ dots \ dots \ 0 \end{pmatrix}$$

• $Gw \leq h$ in cyxopt notation (Julius should supply G and h)

MATRICES IN CVXOPT: RISK PREMIUM

• Risk premium of a portfolio is

$$egin{array}{lll} (\mu_1-r_f & \cdots & \mu_n-r_f) egin{pmatrix} w_1 \ dots \ w_n \end{pmatrix}$$

• $q^{\top}w$ in compared notation (Julius should supply q)

FRONTIER AND TANGENCY PORTFOLIOS

FRONTIER PORTFOLIOS W/O RISK-FREE ASSET

- Frontier \sim hyperbola in mean/std dev space
- Minimize variance subject to expected return = target and weights sum to 1
- Vary target expected return and trace out frontier

TANGENCY PORTFOLIO

- Include risk-free asset
- Minimize variance minus risk premium
- Produces portfolio on capital allocation line
- Divide by sum of weights to get tangency portfolio

EXAMPLE 1: U.S., U.K., FRANCE, GERMANY, AND JAPAN

DATA

- Download international.xlsx and international_corrs.xlsx from the course website.
- Upload the files to Julius. Ask Julius to read them.
- Ask Julius to convert the expected returns, standard deviations, and correlation matrix into separate numpy arrays.
- Ask Julius to compute the covariance matrix from the standard deviations and correlation matrix.

FRONTIER PORTFOLIOS (W/O RISK-FREE ASSET)

- Ask Julius to use cvxopt to minimize variance subject to achieving a target expected return.
- Tell Julius whether you want to allow short sales. To allow short sales, tell Julius there are no inequality constraints.
- Ask Julius to repeat for a range of target expected returns and to plot the expected returns and standard deviations.

TANGENCY PORTFOLIO

- Give Julius a number for the risk-free rate.
- Ask Julius to minimize the variance minus the risk premium.
- Tell Julius whether you want to allow short sales.

- Ask Julius to divide by the sum of the weights to compute the tangency portfolio.
- Ask Julius to include the tangency portfolio and the capital allocation line on the previous plot.

EXAMPLE 2: U.S., DEVELOPED, AND Emerging

U.S., DEVELOPED, AND EMERGING

- Start a new chat.
- Upload us_developed_emerging_rets.xlsx and ask Julius to read it.
- Ask Julius to compute the sample means, sample standard deviations and sample correlation matrix as numpy arrays.
- Repeat the frontier and tangency portfolio calculations.

EXAMPLE 3: ETFS

ETFS FROM YAHOO

- Example: ask Julius to use yfinance to get Yahoo adjusted closing prices for
 - SPY = S&P 500
 - VBR = Vanguard small-cap value
 - IEF = Treasury bonds
 - UUP = U.S. dollar bullish
- Ask Julius to downsample prices to end-of-month and compute monthly returns as percent changes in the downsampled prices.

- Ask Julius to compute means, standard deviations, and correlation matrix as numpy arrays.
- Ask Julius to find frontier of risky assets and tangency portfolio as before.